

χ SB in cascading gauge theory plasma

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Abstract

$\mathcal{N} = 1$ supersymmetric $SU(K + P) \times SU(K)$ cascading gauge theory of Klebanov et.al [1,2] undergoes a first-order finite temperature confinement/deconfinement phase transition at $T_c = 0.6141111(3)\Lambda$, where Λ is the strong coupling scale of the theory. The deconfined phase of the theory, with the unbroken chiral symmetry, extends down to $T_u = 0.8749(0)T_c$, where it becomes perturbatively unstable due to the condensation of the hydrodynamic (sound) modes. We show that at $T_{\chi\text{SB}} = 0.882503(0)T_c > T_u$ the deconfined phase of the cascading plasma is perturbatively unstable towards development of the chiral symmetry breaking (χ SB) condensates. We present evidence that the ground state of the cascading plasma for $T < T_{\chi\text{SB}}$ can not be homogeneous and isotropic.

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1 Introduction and Summary

Consider $\mathcal{N} = 1$ four-dimensional supersymmetric $SU(K + P) \times SU(K)$ gauge theory with two chiral superfields A_1, A_2 in the $(K + P, \overline{K})$ representation, and two fields B_1, B_2 in the $(\overline{K + P}, K)$. This gauge theory has two gauge couplings g_1, g_2 associated with two gauge group factors, and a quartic superpotential

$$W \sim \text{Tr} (A_i B_j A_k B_\ell) \epsilon^{ik} \epsilon^{j\ell}. \quad (1.1)$$

When $P = 0$ above theory flows in the infrared to a superconformal fixed point, commonly referred to as Klebanov-Witten (KW) theory [3]. At the IR fixed point KW gauge theory is strongly coupled — the superconformal symmetry together with $SU(2) \times SU(2) \times U(1)$ global symmetry of the theory implies that anomalous dimensions of chiral superfields $\gamma(A_i) = \gamma(B_i) = -\frac{1}{4}$, *i.e.*, non-perturbatively large.

When $P \neq 0$, conformal invariance of the above $SU(K+P) \times SU(K)$ gauge theory is broken. It is useful to consider an effective description of this theory at energy scale μ with perturbative couplings $g_i(\mu) \ll 1$. It is straightforward to evaluate NSVZ beta-functions for the gauge couplings. One finds that while the sum of the gauge couplings does not run

$$\frac{d}{d \ln \mu} \left(\frac{\pi}{g_s} \equiv \frac{4\pi}{g_1^2(\mu)} + \frac{4\pi}{g_2^2(\mu)} \right) = 0, \quad (1.2)$$

the difference between the two couplings is

$$\frac{4\pi}{g_2^2(\mu)} - \frac{4\pi}{g_1^2(\mu)} \sim P [3 + 2(1 - \gamma_{ij})] \ln \frac{\mu}{\Lambda}, \quad (1.3)$$

where Λ is the strong coupling scale of the theory and γ_{ij} is an anomalous dimension of operators $\text{Tr } A_i B_j$. Given (1.3) and (1.2) it is clear that the effective weakly coupled description of $SU(K+P) \times SU(K)$ gauge theory can be valid only in a finite-width energy band centered about μ scale. Indeed, extending effective description both to the UV and to the IR one necessarily encounters strong coupling in one or the other gauge group factor. As explained in [2], to extend the theory past the strongly coupled region(s) one must perform a Seiberg duality [4]. Turns out, in this gauge theory, a Seiberg duality transformation is a self-similarity transformation of the effective description so that $K \rightarrow K - P$ as one flows to the IR, or $K \rightarrow K + P$ as the energy increases. Thus, extension of the effective $SU(K+P) \times SU(K)$ description to all energy scales involves an infinite sequence - a *cascade* - of Seiberg dualities where the rank of the gauge group is not constant along RG flow, but changes with energy according to [5–7]

$$K = K(\mu) \sim 2P^2 \ln \frac{\mu}{\Lambda}, \quad (1.4)$$

at least as $\mu \gg \Lambda$. To see (1.4), note that the rank changes by $\Delta K \sim P$ as $P \Delta (\ln \frac{\mu}{\Lambda}) \sim 1$. Although there are infinitely many duality cascade steps in the UV, there is only a finite number of duality transformations as one flows to the IR (from a given scale μ). The space of vacua of a generic cascading gauge theory was studied in details in [8]. In the simplest case, when $K(\mu)$ is an integer multiple of P , the cascading gauge theory confines in the infrared with a spontaneous breaking of the chiral symmetry [2].

Effective description of the cascading gauge theory in the UV suggests that it must be ultimately defined as a theory with an infinite number of degrees of freedom. If so, an immediate concern is whether such a theory is renormalizable as a four dimensional quantum field theory, *i.e.*, whether a definite prescription can be made for the computation of all gauge invariant correlation functions in the theory. As was pointed out in [2], whenever $g_s K(\mu) \gg 1$, the cascading gauge theory allows for a dual holographic description [9, 10] as type IIB supergravity on a warped deformed conifold with fluxes. The duality is always valid in the UV of the cascading gauge theory; if, in addition, $g_s P \gg 1$ the holographic correspondence is valid in the IR as well. It was shown in [11] that a cascading gauge theory *defined* by its holographic dual as an RG flow of type IIB supergravity on a warped deformed conifold with fluxes is holographically renormalizable as a four dimensional quantum field theory.

In this paper we study the equilibrium properties of the cascading gauge theory at finite temperature¹. At temperatures $T \gg \Lambda$ the cascading plasma is in the deconfined phase with an unbroken chiral symmetry [5, 16, 17]. The temperature-dependent effective rank $K(T)$ of the cascading theory is large, compare to P [11]:

$$\frac{K(T)}{P^2} = \frac{1}{2} \ln \left(\frac{64\pi^4}{81} \times \frac{sT}{\Lambda^4} \right) \quad \Longrightarrow \quad \frac{K(T)}{P^2} \approx 2 \ln \frac{T}{\Lambda}, \quad T \gg \Lambda. \quad (1.5)$$

In (1.5) s is the entropy density of the plasma at equilibrium. To leading order at higher temperature², the pressure \mathcal{P} and the energy density \mathcal{E} are given by [11]

$$\begin{aligned} \frac{\mathcal{P}}{sT} &= \frac{1}{4} \left(1 - \frac{P^2}{K(T)^2} + \mathcal{O} \left(\frac{P^4}{K(T)^2} \right) \right), \\ \frac{\mathcal{E}}{sT} &= \frac{3}{4} \left(1 + \frac{1}{3} \frac{P^2}{K(T)^2} + \mathcal{O} \left(\frac{P^4}{K(T)^2} \right) \right). \end{aligned} \quad (1.6)$$

In addition to stress-energy tensor, the equilibrium state of the cascading plasma is characterized by the expectation values of two dimension-4 operators: $\mathcal{O}_4^{K_0}$ and $\mathcal{O}_4^{p_0}$, a dimension-6 operator \mathcal{O}_6 , and a dimension-8 operator \mathcal{O}_8 — see [11] for details. As one reduces the temperature, the pressure of the cascading plasma decreases, ultimately turning negative below $T_c = 0.6141111(3)\Lambda$ [18]. At this point, cascading plasma undergoes a first-order confinement/deconfinement first transition. As the transition occurs via nucleation of bubbles of the confined phase, it is non-perturbative. The

¹Hydrodynamics of the cascading gauge theory plasma was discussed in [12–15].

²See [14] for the high-temperature expressions to order $\mathcal{O} \left(\frac{P^8}{K(T)^4} \right)$.

deconfined phase of the cascading plasma remains as a metastable phase all the way down to $T_u = 0.8749(0)T_c$, at which point it joins a perturbatively unstable branch of the theory with negative specific heat³, see [14].

The deconfined phase of the cascading plasma extensively studied in [11, 14, 18] does not spontaneously break chiral $U(1)$ symmetry. The latter is obvious by the absence of the expectation values for dimension-3 operators in the studied thermal states. On the other hand, the zero-temperature supersymmetric ground state of the theory spontaneously breaks chiral symmetry $U(1) \supset \mathbb{Z}_2$ [2]. The question we would like to address in this paper is whether or not spontaneous symmetry breaking occurs in the deconfined phase of the cascading plasma. We emphasize *spontaneous* symmetry breaking as opposite to considering thermal states of the mass-deformed cascading gauge theory. It is fairly straightforward to study mass-deformed cascading gauge theory. In the latter case, one introduces the mass terms

$$\mu_i \equiv \frac{m_i}{\Lambda}, \quad i = 1, 2, \quad (1.7)$$

for the gauginos ($\mathcal{N} = 1$ fermionic superpartners of $SU(K + P) \times SU(K)$ gauge bosons). These mass terms explicitly break both the supersymmetry and the chiral $U(1)$ symmetry. As we show in section 5, it is straightforward to construct homogeneous and isotropic thermal states of the mass-deformed cascading plasma. Necessarily, these states have nonzero expectation value for dimension-3 operators

$$\mathcal{O}_3^j = \mathcal{O}_3^j(\mu_i), \quad j = 1, 2, \quad (1.8)$$

(gaugino bilinear condensates of the two gauge group factors). We show that in the chiral limit $\mu_i \rightarrow 0$, the condensates vanish as well:

$$\lim_{\mu_i \rightarrow 0} \mathcal{O}_3^j(\mu_i) = 0. \quad (1.9)$$

Naively, the statement (1.9) would imply that the deconfined cascading plasma does not break chiral symmetry. We argue in section 3 that this is not the case. Specifically, we carefully study physical excitations in the cascading plasma, responsible for the development of the chiral condensates, and show that these fluctuations become tachyonic at temperatures $T < T_{\chi\text{SB}} = 0.882503(0)T_c$. Thus, they must condense. The vanishing of the homogeneous condensates in the chiral limit (1.9) strongly suggests

³Critical phenomena in the cascading plasma in the vicinity of T_u was discussed in [19]. Further analysis of the relevant critical universality class were performed in [20].

that the 'chiral tachyons' we discover in section 3 condense with a finite momentum — the resulting ground state can not be homogeneous and isotropic. We present a further (technical) evidence for the latter in section 4.

A more detailed outline of the rest of the paper follows. In section 2 we present consistent truncation of type IIB supergravity on warped deformed conifold with fluxes and $SU(2) \times SU(2) \times \mathbb{Z}_2$ global symmetry. The resulting five-dimensional effective gravitational action, which we refer to as a 'KS effective action', is dual to a strongly coupled cascading gauge theory. In section 2.1 we discuss further truncation of the KS effective action to the Klebanov-Tseytlin (KT) effective action derived in [11]. In section 2.2 we derive equations of motion for the homogeneous and isotropic states of the cascading gauge theory (at zero or non-zero temperatures) and recover supersymmetric KT solution [1], supersymmetric KS solution [2], and the gravitational solutions describing the deconfined chirally symmetric phase of the cascading plasma [11,18]. In section 2.3 we derive the effective action for the linearized fluctuations dual to chiral condensates about chirally-symmetric states of the cascading theory. In section 3 we compute the spectrum of quasinormal modes of the chiral fluctuations described by the effective action of section 2.2 about the deconfined chirally-symmetric states of the cascading plasma. We show that for temperatures $T < T_{\chi\text{SB}} = 0.882503(0)T_c$ these quasinormal modes realize Gregory-Laflamme instability of the translationary invariant Klebanov-Tseytlin horizons of [18]. Since the deconfined cascading plasma is thermodynamically stable down to T_u , and $T_{\chi\text{SB}} > T_u$, the gravitational dual to cascading gauge theory plasma presents an interesting string-theoretic example of violation of the correlated stability conjecture (CSC) [21,22]⁴. In section 4 we discuss the gravitational solutions describing the homogeneous and isotropic states of the cascading plasma with spontaneously broken chiral symmetry. We attempt (unsuccessfully) to construct these solutions by deforming chirally symmetric states of the cascading plasma for $T < T_{\chi\text{SB}}$ along the tachyonic directions discovered in section 3. Finally, in section 5 we construct homogeneous and isotropic gravitational solutions dual to equilibrium states of mass-deformed cascading plasma for $T < T_{\chi\text{SB}}$. Constructed thermal states explicitly break chiral symmetry. We show that in the chiral limit these homogeneous and isotropic states do not break chiral symmetry spontaneously, see (1.9).

⁴See [23] for a recent discussion of CSC.

2 KS effective action

We take a perspective of [11] where the cascading gauge theory at strong coupling is defined via its holographic dual, *i.e.*, by type IIB string theory on warped deformed conifold with fluxes and $SU(2) \times SU(2) \times \mathbb{Z}_2$ global symmetry. We begin with deriving an effective five-dimensional gravitational action representing the holographic dual of the cascading gauge theory.

We will work in the gravitational approximation to type IIB string theory, using the type IIB supergravity action. This action takes the form (in the Einstein frame)

$$S_{10} = \frac{1}{16\pi G_{10}} \int_{\mathcal{M}_{10}} \left(R_{10} \wedge \star 1 - \frac{1}{2} d\Phi \wedge \star d\Phi - \frac{1}{2} e^{-\Phi} H_3 \wedge \star H_3 - \frac{1}{2} e^{\Phi} F_3 \wedge \star F_3 - \frac{1}{4} F_5 \wedge \star F_5 - \frac{1}{2} C_4 \wedge H_3 \wedge F_3 \right), \quad (2.1)$$

where \mathcal{M}_{10} is the ten dimensional bulk space-time, G_{10} is the ten dimensional gravitational constant, and we have consistently set the axion C_0 to zero (it vanishes in all the solutions we are interested in). In this action

$$F_3 = dC_2, \quad F_5 = dC_4 - C_2 \wedge H_3, \quad (2.2)$$

where C_2 and C_4 are the Ramond-Ramond (RR) potentials. The equations of motion following from the action (2.1) have to be supplemented by the self-duality condition

$$\star F_5 = F_5. \quad (2.3)$$

It is important to remember that the self-duality condition (2.3) can not be imposed at the level of the action, as this would lead to wrong equations of motion.

Introduce the following 1-forms on $T^{1,1}$ [2]:

$$\begin{aligned} g_1 &= \frac{\alpha^1 - \alpha^3}{\sqrt{2}}, & g_2 &= \frac{\alpha^2 - \alpha^4}{\sqrt{2}}, \\ g_3 &= \frac{\alpha^1 + \alpha^3}{\sqrt{2}}, & g_4 &= \frac{\alpha^2 + \alpha^4}{\sqrt{2}}, \\ g_5 &= \alpha^5, \end{aligned} \quad (2.4)$$

where

$$\begin{aligned} \alpha^1 &= -\sin \theta_1 d\phi_1, & \alpha^2 &= d\theta_1, \\ \alpha^3 &= \cos \psi \sin \theta_2 d\phi_2 - \sin \psi d\theta_2, \\ \alpha^4 &= \sin \psi \sin \theta_2 d\phi_2 + \cos \psi d\theta_2, \\ \alpha^5 &= d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2. \end{aligned} \quad (2.5)$$

The Einstein-frame metric ansatz is

$$ds_{10}^2 = g_{\mu\nu}(y)dy^\mu dy^\nu + \Omega_1^2(y)g_5^2 + \Omega_2^2(y)[g_3^2 + g_4^2] + \Omega_3^2(y)[g_1^2 + g_2^2], \quad (2.6)$$

where y denotes the coordinates of \mathcal{M}_5 (greek indices μ, ν will run from 0 to 4). Additionally, we assume the following ansatz for the fluxes $H_3 \equiv dB_2$, F_3 and the dilaton Φ :

$$\begin{aligned} B_2 &= h_1(y) g_1 \wedge g_2 + h_3(y) g_3 \wedge g_4, \\ F_3 &= \frac{1}{9}P g_5 \wedge g_3 \wedge g_4 + h_2(y) (g_1 \wedge g_2 - g_3 \wedge g_4) \wedge g_5 \\ &\quad + (g_1 \wedge g_3 + g_2 \wedge g_4) \wedge d(h_2(y)), \\ \Phi &= \Phi(y), \end{aligned} \quad (2.7)$$

where P is an integer corresponding to the RR 3-form flux on the compact 3-cycle (and to the number of fractional branes on the conifold). Special care should be taken with the RR 5-form. From (2.2) we get the Bianchi identity

$$dF_5 = -F_3 \wedge H_3, \quad (2.8)$$

which for the background fluxes (2.7) is solved by

$$F_5 = dC_4 + \left(4\Omega_0 + h_2(y)(h_3(y) - h_1(y)) + \frac{1}{9}Ph_1(y) \right) g_5 \wedge g_3 \wedge g_4 \wedge g_1 \wedge g_2, \quad (2.9)$$

with some constant Ω_0 . In our ansatz the RR four-form does not depend on the compact coordinates, that is $C_4 \equiv C_4(y)$ (note that $C_4 \wedge F_3 \wedge H_3 \neq 0$), and the RR five-form is proportional to the volume form of \mathcal{M}_5 (plus its dual). We define $F(y)$ by

$$dC_4 = \frac{F(y)}{\Omega_1\Omega_2\Omega_3^2} \text{vol}_{\mathcal{M}_5} \equiv \frac{F(y)}{\Omega_1\Omega_2\Omega_3^2} \sqrt{-\det(g_{\mu\nu})} dy^1 \wedge \dots \wedge dy^5, \quad (2.10)$$

and then the self-duality condition (2.3) implies

$$F(y) = 4\Omega_0 + h_2(y)(h_3(y) - h_1(y)) + \frac{1}{9}Ph_1(y), \quad (2.11)$$

(again, in deriving the effective action we should keep C_4 unconstrained and impose this equation later). Altogether, from the five-dimensional perspective we allow fluctuations in the metric $g_{\mu\nu}(y)$, in the scalar fields $\Omega_1(y), \Omega_2(y), \Omega_3(y), h_1(y), h_2(y), h_3(y), \Phi(y)$ and in the four-form $C_4(y)$ (which is determined in terms of the others by the self-duality condition). We have set to zero various fluctuations of the form fields which

are p -forms on \mathcal{M}_5 , and also fluctuations of C_2 of the same form as the fluctuation of B_2 in (2.7), even though they are allowed by the symmetries. This is a consistent truncation of the full ten dimensional supergravity action.

We now perform the KK reduction of (2.1) by plugging into it the ansatz described above. Recall that

$$\text{vol}_{T^{1,1}} \equiv \frac{1}{108} \int g_5 \wedge g_3 \wedge g_4 \wedge g_1 \wedge g_2 = \frac{1}{108} \times \frac{16\pi^3}{27}. \quad (2.12)$$

First, we have

$$\int_{\mathcal{M}_{10}} 1 \wedge \star 1 = 108 \text{vol}_{T^{1,1}} \int_{\mathcal{M}_5} \Omega_2 \Omega_2^2 \Omega_3^2 \text{vol}_{\mathcal{M}_5}. \quad (2.13)$$

With a straightforward but somewhat tedious computation we find that in the background (2.6)

$$\begin{aligned} R_{10} = R_5 &+ \left(\frac{1}{2\Omega_1^2} + \frac{2}{\Omega_2^2} + \frac{2}{\Omega_3^2} - \frac{\Omega_2^2}{4\Omega_1^2\Omega_3^2} - \frac{\Omega_3^2}{4\Omega_1^2\Omega_2^2} - \frac{\Omega_1^2}{\Omega_2^2\Omega_3^2} \right) - 2\Box \ln(\Omega_1\Omega_2^2\Omega_3^2) \\ &- \left\{ (\nabla \ln \Omega_1)^2 + 2(\nabla \ln \Omega_2)^2 + 2(\nabla \ln \Omega_3)^2 + (\nabla \ln(\Omega_1\Omega_2^2\Omega_3^2))^2 \right\}, \end{aligned} \quad (2.14)$$

where R_5 is the five dimensional Ricci scalar of the metric

$$ds_5^2 = g_{\mu\nu}(y) dy^\mu dy^\nu. \quad (2.15)$$

In (2.14), ∇_λ denotes the covariant derivative with respect to the metric (2.15), explicitly given by

$$\begin{aligned} \nabla_\lambda \Omega_i &= \partial_\lambda \Omega_i, \\ \nabla_\lambda \nabla_\nu \Omega_i &= \partial_\lambda \partial_\nu \Omega_i - \Gamma_{\lambda\nu}^\rho \partial_\rho \Omega_i. \end{aligned} \quad (2.16)$$

Now, by plugging our ansatz into (2.1) we find that it reduces to the following effective action :

$$\begin{aligned} S_5 = & \frac{108}{16\pi G_5} \int_{\mathcal{M}_5} \text{vol}_{\mathcal{M}_5} \Omega_1 \Omega_2^2 \Omega_3^2 \left\{ R_{10} - \frac{1}{2} (\nabla \Phi)^2 \right. \\ & - \frac{1}{2} e^{-\Phi} \left(\frac{(h_1 - h_3)^2}{2\Omega_1^2 \Omega_2^2 \Omega_3^2} + \frac{1}{\Omega_3^4} (\nabla h_1)^2 + \frac{1}{\Omega_2^4} (\nabla h_3)^2 \right) \\ & - \frac{1}{2} e^{\Phi} \left(\frac{2}{\Omega_2^2 \Omega_3^2} (\nabla h_2)^2 + \frac{1}{\Omega_1^2 \Omega_2^4} \left(h_2 - \frac{P}{9} \right)^2 + \frac{1}{\Omega_1^2 \Omega_3^4} h_2^2 \right) \\ & \left. - \frac{1}{4} \left(\frac{F^2}{\Omega_1^2 \Omega_2^4 \Omega_3^4} + \frac{5}{24} \mathcal{F}_{\mu_1 \dots \mu_5} \mathcal{F}^{\mu_1 \dots \mu_5} \right) \right\} \\ & + \frac{108}{16\pi G_5} \frac{1}{2} \int_{\mathcal{M}_5} dF \wedge C_4, \end{aligned} \quad (2.17)$$

where

$$\mathcal{F}_{\mu_1 \dots \mu_5} \equiv \partial_{[\mu_1} C_{4 \mu_2 \dots \mu_5]} = \frac{1}{5} \frac{F}{\Omega_1 \Omega_2^2 \Omega_3^2} \sqrt{-\det(g_{\mu\nu})} \epsilon_{\mu_1 \dots \mu_5}, \quad (2.18)$$

($[\dots]$ denotes anti-symmetrization with weight one) and G_5 is the five dimensional effective gravitational constant

$$G_5 \equiv \frac{G_{10}}{\text{vol}_{T^{1,1}}}. \quad (2.19)$$

Note that our gravitational action is not the standard five dimensional action because of the factor of $\Omega_1 \Omega_2^2 \Omega_3^2$ in front of the five dimensional Einstein-Hilbert term.

In the five dimensional action it turns out to be possible to “integrate out” the field C_4 using the self-duality equation (2.11) and to obtain an action involving only the other fields. This leads to the action we will be using in this paper

$$\begin{aligned} S_5 = & \frac{108}{16\pi G_5} \int_{\mathcal{M}_5} \text{vol}_{\mathcal{M}_5} \Omega_1 \Omega_2^2 \Omega_3^2 \left\{ R_{10} - \frac{1}{2} (\nabla \Phi)^2 \right. \\ & - \frac{1}{2} e^{-\Phi} \left(\frac{(h_1 - h_3)^2}{2\Omega_1^2 \Omega_2^2 \Omega_3^2} + \frac{1}{\Omega_3^4} (\nabla h_1)^2 + \frac{1}{\Omega_2^4} (\nabla h_3)^2 \right) \\ & - \frac{1}{2} e^{\Phi} \left(\frac{2}{\Omega_2^2 \Omega_3^2} (\nabla h_2)^2 + \frac{1}{\Omega_1^2 \Omega_2^4} \left(h_2 - \frac{P}{9} \right)^2 + \frac{1}{\Omega_1^2 \Omega_3^4} h_2^2 \right) \\ & \left. - \frac{1}{2\Omega_1^2 \Omega_2^4 \Omega_3^4} \left(4\Omega_0 + h_2 (h_3 - h_1) + \frac{1}{9} P h_1 \right)^2 \right\}, \end{aligned} \quad (2.20)$$

where R_{10} is given by (2.14).

2.1 Reduction of KS effective action to KT effective action

Effective action (2.20) allows for further consistent truncation. Indeed, setting

$$\begin{aligned} ds_5^2 &= (ds_5^2)^{KT}, & \Omega_1 &= \frac{1}{3} \Omega_1^{KT}, & \Omega_2 &= \Omega_3 = \frac{1}{\sqrt{6}} \Omega^{KT}, & F &= \frac{K^{KT}}{108}, \\ h_1 = h_3 &= \frac{1}{6} \tilde{k}^{KT}, & h_2 &= \frac{P}{18}, & \Phi &= \Phi^{KT}, & \Omega_0 &= \frac{\tilde{K}_0^{KT}}{432}, \end{aligned} \quad (2.21)$$

we obtain effective action of [11] describing $SU(2) \times SU(2) \times U(1)$ symmetric states of the cascading gauge theory. In (2.21) we used superscript ‘KT’ to relate to fields of the KT effective action in [11].

2.2 Homogeneous and isotropic $SU(2) \times SU(2) \times \mathbb{Z}_2$ states of the cascading gauge theory

In this section we derive gravitational equations of motion from the effective action (2.20) describing homogeneous and isotropic states of the cascading gauge theory at zero and nonzero temperature. In the latter case the background geometry has a regular (homogeneous and isotropic) Schwartzchild horizon. We recover from the obtained equations of motion supersymmetric Klebanov-Tseytlin [2] and Klebanov-Strassler [2] solutions, as well as KT BH solution of [18].

The general five-dimensional background geometry with homogeneous and isotropic (but not necessary Lorentz-invariant) asymptotic boundary takes form

$$ds_5^2 = H^{-1/2} (-f_1^2 dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + H^{1/2} \omega_1^2 \frac{dr^2}{\tilde{f}_2^2}, \quad (2.22)$$

$$\Omega_i = \omega_i H^{1/4}, \quad g_s = e^\Phi,$$

where $H = H(r)$, $f_1 = f_1(r)$, $\tilde{f}_2 = \tilde{f}_2(r)$ and $\omega_i = \omega_i(r)$. Additionally, we set $h_i = h_i(r)$ and $g_s = g_s(r)$.

From (2.20) we find the following equations of motion⁵

$$0 = h_1'' + h_1' \left[\ln \frac{f_1 \tilde{f}_2 \omega_2^2}{\omega_3^2 H g_s} \right]' - h_1 \left(\frac{g_s (P - 9h_2)^2}{81 H \tilde{f}_2^2 \omega_2^4} + \frac{\omega_3^2}{2 \tilde{f}_2^2 \omega_2^2} \right) + \frac{(h_2 h_3 + 4\Omega_0)(9h_2 - P)g_s}{9 H \tilde{f}_2^2 \omega_2^4} + \frac{\omega_3^2 h_3}{2 \omega_2^2 \tilde{f}_2^2}, \quad (2.23)$$

$$0 = h_2'' + h_2' \left[\ln \frac{f_1 \tilde{f}_2 g_s}{H} \right]' - h_2 \left(\frac{\omega_2^4 + \omega_3^4}{2 \tilde{f}_2^2 \omega_3^2 \omega_2^2} + \frac{(h_1 - h_3)^2}{2 H \tilde{f}_2^2 \omega_2^2 \omega_3^2 g_s} \right) + \frac{(h_1 - h_3)(P h_1 + 36\Omega_0)}{18 H \tilde{f}_2^2 \omega_2^2 \omega_3^2 g_s} + \frac{\omega_3^2 P}{18 \tilde{f}_2^2 \omega_2^2}, \quad (2.24)$$

$$0 = h_3'' + h_3' \left[\ln \frac{f_1 \tilde{f}_2 \omega_3^2}{\omega_2^2 H g_s} \right]' - \frac{h_3 (2h_2^2 g_s + \omega_2^2 H \omega_3^2)}{2 H \tilde{f}_2^2 \omega_3^4} + \frac{g_s (h_1 h_2 - 4\Omega_0)(9h_2 - P)}{9 H \tilde{f}_2^2 \omega_3^4} + \frac{\omega_2^2 h_1}{2 \tilde{f}_2^2 \omega_3^2} - \frac{4g_s \Omega_0 P}{9 H \tilde{f}_2^2 \omega_3^4}, \quad (2.25)$$

⁵We verified that exactly the same equations of motion arise directly from type IIB supergravity in ten dimensions.

$$0 = g_s'' - \frac{(g_s')^2}{g_s} + g_s' \left[\ln f_1 \tilde{f}_2 \omega_2^2 \omega_3^2 \right]' - \frac{g_s^2}{H} \left(\frac{(h_2')^2}{\omega_2^2 \omega_3^2} + \frac{h_2^2}{2 \tilde{f}_2^2 \omega_3^4} + \frac{(9h_2 - P)^2}{162 \tilde{f}_2^2 \omega_2^4} \right) + \frac{1}{2H} \left(\frac{(h_1')^2}{\omega_3^4} + \frac{(h_3')^2}{\omega_2^4} \right) + \frac{(h_3 - h_1)^2}{4 \omega_2^2 \omega_3^2 \tilde{f}_2^2 H}, \quad (2.26)$$

$$0 = f_1'' + f_1' \left[\ln \tilde{f}_2 \omega_2^2 \omega_3^2 \right]', \quad (2.27)$$

$$0 = H'' - \frac{(H')^2}{H} + H' \left[\ln f_1 \tilde{f}_2 \omega_2^2 \omega_3^2 \right]' + \frac{(9h_2(h_3 - h_1) + Ph_1 + 36\Omega_0)^2}{81 \tilde{f}_2^2 \omega_2^4 \omega_3^4 H} + \frac{1}{2g_s} \left(\frac{(h_1')^2}{\omega_3^4} + \frac{(h_3')^2}{\omega_2^4} \right) + \frac{g_s(h_2')^2}{\omega_3^2 \omega_2^2} + \frac{(h_3 - h_1)^2}{4 \tilde{f}_2^2 \omega_2^2 \omega_3^2 g_s} + \frac{g_s h_2^2}{2 \tilde{f}_2^2 \omega_3^4} + \frac{g_s(9h_2 - P)^2}{162 \tilde{f}_2^2 \omega_2^4}, \quad (2.28)$$

$$0 = \omega_1'' - \frac{(\omega_1')^2}{\omega_1} + \omega_1' \left[\ln f_1 \tilde{f}_2 \omega_2^2 \omega_3^2 \right]' - \frac{\omega_1}{4H g_s} \left(\frac{(h_1')^2}{\omega_3^4} + \frac{(h_3')^2}{\omega_2^4} \right) - \frac{\omega_1 g_s (h_2')^2}{2 \omega_3^2 \omega_2^2 H} + \frac{\omega_1 ((\omega_2^2 - \omega_3^2)^2 - 4\omega_1^4)}{4 \omega_2^2 \omega_3^2 \tilde{f}_2^2} + \frac{\omega_1}{H} \left(\frac{(h_3 - h_1)^2}{8 \tilde{f}_2^2 \omega_2^2 \omega_3^2 g_s} + \frac{g_s h_2^2}{4 \omega_3^4 \tilde{f}_2^2} + \frac{g_s(9h_2 - P)^2}{324 \tilde{f}_2^2 \omega_2^4} \right), \quad (2.29)$$

$$0 = \omega_2'' + \frac{(\omega_2')^2}{\omega_2} + \omega_2' \left[\ln f_1 \tilde{f}_2 \omega_3^2 \right]' - \frac{\omega_2}{4H g_s} \left(\frac{(h_1')^2}{\omega_3^4} - \frac{(h_3')^2}{\omega_2^4} \right) - \frac{g_s(81h_2^2 \omega_2^4 - \omega_3^4(9h_2 - P)^2)}{324 \omega_2^3 \omega_3^4 \tilde{f}_2^2 H} - \frac{(\omega_2^4 - \omega_3^4 + 8\omega_3^2 \omega_1^2 - 4\omega_1^4)}{8 \omega_3^2 \omega_2 \tilde{f}_2^2}, \quad (2.30)$$

$$0 = \omega_3'' + \frac{(\omega_3')^2}{\omega_3} + \omega_3' \left[\ln f_1 \tilde{f}_2 \omega_2^2 \right]' + \frac{\omega_3}{4H g_s} \left(\frac{(h_1')^2}{\omega_3^4} - \frac{(h_3')^2}{\omega_2^4} \right) + \frac{g_s(81h_2^2 \omega_2^4 - \omega_3^4(9h_2 - P)^2)}{324 \omega_3^3 \omega_2^4 \tilde{f}_2^2 H} - \frac{\omega_3^4 - \omega_2^4 + 8\omega_2^2 \omega_1^2 - 4\omega_1^4}{8 \omega_3 \omega_2^2 \tilde{f}_2^2}, \quad (2.31)$$

$$0 = ([\ln g_s]')^2 + ([\ln H]')^2 + \frac{1}{H g_s} \left(\frac{(h_1')^2}{\omega_3^4} + \frac{(h_3')^2}{\omega_2^4} \right) + 2 \frac{g_s(h_2')^2}{H \omega_3^2 \omega_2^2} - 4 ([\ln \omega_2 \omega_3]')^2 - 8 [\ln \omega_3]' [\ln \omega_1 \omega_2 f_1]' - [\ln f_1]' [\ln H^2 \omega_1^4 \omega_2^8]' - 8 [\ln \omega_1]' [\ln \omega_2]' - \frac{(9h_2(h_3 - h_1) + Ph_1 + 36\Omega_0)^2}{81 \omega_2^4 \tilde{f}_2^2 \omega_3^4 H^2} - \frac{(\omega_2^2 - \omega_3^2)^2 - 4\omega_1^2(2\omega_2^2 - \omega_1^2 + 2\omega_3^2)}{2 \omega_3^2 \omega_2^2 \tilde{f}_2^2} - \frac{1}{H \tilde{f}_2^2} \left(\frac{(h_1 - h_3)^2}{2 \omega_2^2 g_s \omega_3^2} + \frac{g_s(9h_2 - P)^2}{81 \omega_2^4} + \frac{g_s h_2^2}{\omega_3^4} \right). \quad (2.32)$$

We explicitly verified that the constraint (2.32) associated with the reparametrization of the radial coordinate r is consistent with the second order equations of motion (2.23)-(2.31).

2.2.1 Supersymmetric KT solution

The singular KT solution [1] to (2.23)-(2.32) is given by

$$\begin{aligned} h_1 = h_3 &= -\frac{1}{6}P g_0 \ln r, & h_2 &= \frac{P}{18}, & f_1 &= 1, & \tilde{f}_2 &= \frac{r}{3}, & g_s &= g_0 \\ \omega_1 &= \frac{1}{3r}, & \omega_2 = \omega_3 &= \frac{1}{\sqrt{6}r}, & H &= r^4 \left(108\Omega_0 + \frac{1}{8}P^2 g_0 - \frac{1}{2}P^2 g_0 \ln r \right) \end{aligned} \quad (2.33)$$

where $r \rightarrow 0$ is the boundary.

2.2.2 Supersymmetric KS solution

The supersymmetric KS solution [2] to (2.23)-(2.32) is given by

$$\begin{aligned} h_1 &= \frac{P g_0 (\cosh r - 1)}{18 \sinh r} \left(\frac{r \cosh r}{\sinh r} - 1 \right), & h_2 &= \frac{P}{18} \left(1 - \frac{r}{\sinh r} \right), \\ h_3 &= \frac{P g_0 (\cosh r + 1)}{18 \sinh r} \left(\frac{r \cosh r}{\sinh r} - 1 \right), & f_1 = \tilde{f}_2 &= 1, & g_s &= g_0, \\ \omega_1 &= \frac{\epsilon^{2/3}}{\sqrt{6}\hat{K}}, & \omega_2 &= \frac{\epsilon^{2/3}\hat{K}^{1/2}}{\sqrt{2}} \cosh \frac{r}{2}, & \omega_3 &= \frac{\epsilon^{2/3}\hat{K}^{1/2}}{\sqrt{2}} \sinh \frac{r}{2}, \end{aligned} \quad (2.34)$$

with

$$\hat{K} = \frac{(\sinh(2r) - 2r)^{1/3}}{2^{1/3} \sinh r}, \quad H' = \frac{16((9h_2 - P)h_1 - 9h_3 h_2)}{9\epsilon^{8/3}\hat{K}^2 \sinh^2 r}, \quad \Omega_0 = 0, \quad (2.35)$$

where now $r \rightarrow \infty$ is the boundary.

2.2.3 KT BH solution

The KT BH equations of motion in the parametrization of [18] are obtained from (2.23)-(2.32) introducing a radial coordinate

$$x \equiv 1 - f_1(r), \quad (2.36)$$

and setting

$$\begin{aligned} h_1 = h_3 &= \frac{1}{P} \left(\frac{K}{12} - 36\Omega_0 \right), & h_2(x) &= \frac{P}{18}, & g_s(x) &= g, \\ \omega_1(x) &= \frac{f_2^{1/2}}{3(2x - x^2)^{1/4}}, & \omega_2 = \omega_3 &= \frac{f_3^{1/2}}{\sqrt{6}(2x - x^2)^{1/4}}, \\ H(x) &= (2x - x^2)h. \end{aligned} \quad (2.37)$$

2.3 Chiral fluctuations in cascading plasma

Recall that unlike (2.20), the effective action obtained with further consistent truncation

$$h_1 = h_3, \quad h_2 = \frac{P}{18}, \quad \Omega_2 = \Omega_3, \quad (2.38)$$

has an enlarged global symmetry, *i.e.*, \mathbb{Z}_2 get enhanced to $U(1)$. On the dual gauge theory side such enhancement corresponds to restoration of the chiral symmetry. As familiar from [2], the chiral symmetry of the cascading theory is spontaneously broken at a supersymmetric ground state. In this section we compute effective gravitational action of the linearized fluctuations about chirally symmetric states of the cascading gauge theory.

We introduce

$$\begin{aligned} h_1 &= \frac{1}{P} \left(\frac{K_1}{12} - 36\Omega_0 \right), & h_2 &= \frac{P}{18} K_2, & h_3 &= \frac{1}{P} \left(\frac{K_3}{12} - 36\Omega_0 \right), \\ \Omega_1 &= \frac{1}{3} f_c^{1/2} h^{1/4}, & \Omega_2 &= \frac{1}{\sqrt{6}} f_a^{1/2} h^{1/4}, & \Omega_3 &= \frac{1}{\sqrt{6}} f_b^{1/2} h^{1/4}. \end{aligned} \quad (2.39)$$

It is straightforward to verify that linearized fluctuations $\{\delta f, \delta k_1, \delta k_2\}$ in

$$\begin{aligned} K_1 &= K + \delta k_1, & K_2 &= 1 + \delta k_2, & K_3 &= K - \delta k_1, \\ f_c &= f_2, & f_a &= f_3 + \delta f, & f_b &= f_3 - \delta f, \end{aligned} \quad (2.40)$$

decouple from all the other fluctuations, provided the gravitational fields

$$\left\{ ds_5^2, K, h, f_2, f_3, g_s \right\} \quad (2.41)$$

are on-shell, *i.e.*, describe a chirally symmetric state of the cascading plasma. The effective action for the χ SB fluctuations can be derived from (2.20):

$$S_{\chi\text{SB}}[\delta f, \delta k_1, \delta k_2] = \frac{1}{16\pi G_5} \int_{\mathcal{M}_5} \text{vol}_{\mathcal{M}_5} h^{5/4} f_2^{1/2} f_3^2 \left\{ \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 \right\}, \quad (2.42)$$

$$\mathcal{L}_1 = -\frac{(\delta f)^2}{f_3^2} \left(-\frac{P^2 e^\Phi}{2f_2 h^{3/2} f_3^2} - \frac{(\nabla K)^2}{8f_3^2 h P^2 e^\Phi} - \frac{K^2}{2f_2 h^{5/2} f_3^2} \right), \quad (2.43)$$

$$\begin{aligned} \mathcal{L}_2 &= -\frac{9f_3^2 - 24f_2 f_3 + 4f_2^2}{f_2 h^{1/2} f_3^4} (\delta f)^2 + 2\Box \frac{(\delta f)^2}{f_3^2} - \left(\nabla \frac{(\delta f)^2}{f_3^2} \right)^2 \\ &\quad - 2\nabla \left(\ln h^{1/4} f_3^{1/2} \right) \nabla \left(\frac{(\delta f)^2}{f_3^2} \right) + 2\nabla \left(\ln f_2^{1/2} h^{5/4} f_3^2 \right) \nabla \left(\frac{(\delta f)^2}{f_3^2} \right), \end{aligned} \quad (2.44)$$

$$\mathcal{L}_3 = -\frac{1}{2P^2 e^\Phi} \left(\frac{9}{2f_2 h^{3/2} f_3^2} (\delta k_1)^2 + \frac{1}{2h f_3^4} \left(2(\nabla K)^2 (\delta f)^2 + f_3^2 (\nabla \delta k_1)^2 + 4f_3 \delta f \nabla K \nabla \delta k_1 \right) \right), \quad (2.45)$$

$$\mathcal{L}_4 = \frac{P^2 e^\Phi}{2} \left(\frac{2}{9h f_3^2} (\nabla \delta k_2)^2 + \frac{2}{f_2 h^{3/2} f_3^4} (3(\delta f)^2 + 4f_3 \delta f \delta k_2 + f_3^3 (\delta k_2)^2) \right), \quad (2.46)$$

$$\mathcal{L}_5 = \frac{K}{f_2 h^{5/2} f_3^6} (f_3^2 \delta k_1 \delta k_2 - K (\delta f)^2). \quad (2.47)$$

3 χ SB quasinormal modes of the KT BH

In this section we study the spectrum of the χ SB quasinormal modes of Klebanov-Tseytlin black hole [14, 18] and show that these modes are unstable (tachyonic) once $T < T_{\chi\text{SB}}$, with

$$T_{\chi\text{SB}} = 0.882503(0) T_c, \quad (3.1)$$

where T_c is the critical temperature of the first-order confinement deconfinement phase transition. Although the spontaneous breaking of the chiral symmetry occurs in the meta-stable deconfined phase of the cascading plasma, this perturbative instability precedes the hydrodynamic instability in of the deconfined phase discovered in [14] since $T_{\chi\text{SB}} > T_u = 0.8749(0)T_c$.

Effective action describing the χ SB fluctuations in cascading plasma is given by (2.42)-(2.47). The background geometry dual to the deconfined homogeneous and isotropic phase of the cascading plasma is given by (see (2.22) and (2.39))

$$ds_5^2 = h^{-1/2} (1 - f_1^2)^{-1/2} (-f_1^2 dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{1}{3} h^{1/2} f_2 \frac{dr^2}{\tilde{f}_2^2}, \quad (3.2)$$

with $\{f_1, \tilde{f}_2, K, h, f_2, f_3, g_s\}$ being functions of r only. Without loss of generality we assume

$$\delta f = e^{-i\omega t + i k x_3} F, \quad \delta k_1 = e^{-i\omega t + i k x_3} \mathcal{K}_1, \quad \delta k_2 = e^{-i\omega t + i k x_3} \mathcal{K}_2, \quad (3.3)$$

where $\{F, \mathcal{K}_1, \mathcal{K}_2\}$ are functions of the radial coordinate only, satisfying the following

equations of motion

$$\begin{aligned}
0 = & -\omega^2 F + f_1^2 k^2 F - \frac{9\tilde{f}_2^2 f_1^2}{h(1-f_1^2)^{1/2} f_2} F'' \\
& + \frac{9\tilde{f}_2 f_1}{h(1-f_1^2)^{3/2} f_2} (f_1^3 \tilde{f}_2' - \tilde{f}_2 f_1' f_1^2 - f_1 \tilde{f}_2' - \tilde{f}_2 f_1') F' \\
& + \frac{9f_1^2 K' \tilde{f}_2^2}{2(1-f_1^2)^{1/2} h^2 g_s f_2 f_3} \mathcal{K}_1' - \frac{f_1^2}{2(1-f_1^2)^{5/2} f_2 g_s h^2 f_3^2} \left(-18f_3^2 h g_s f_1^4 - 4g_s^2 f_1^4 \right. \\
& + 18h g_s \tilde{f}_2^2 f_1^4 (f_3')^2 - 9\tilde{f}_2^2 f_1^4 (K')^2 + 8f_1^4 h g_s f_2^2 + 36f_3^2 h g_s f_1^2 + 8g_s^2 f_1^2 \\
& - 36h g_s \tilde{f}_2^2 f_1^2 (f_3')^2 + 18\tilde{f}_2^2 f_1^2 (K')^2 - 16f_1^2 h g_s f_2^2 + 8h g_s f_2^2 - 4g_s^2 + 36f_3^2 h g_s \tilde{f}_2^2 (f_1')^2 \\
& \left. - 9\tilde{f}_2^2 (K')^2 + 18h g_s \tilde{f}_2^2 (f_3')^2 - 18f_3^2 h g_s \right) F + \frac{2f_1^2 g_s}{(1-f_1^2)^{1/2} h^2 f_2 f_3} \mathcal{K}_2, \tag{3.4}
\end{aligned}$$

$$\begin{aligned}
0 = & -\omega^2 \mathcal{K}_1 + f_1^2 k^2 \mathcal{K}_1 - \frac{9\tilde{f}_2^2 f_1^2}{h(1-f_1^2)^{1/2} f_2} \mathcal{K}_1'' \\
& - \frac{9f_1 \tilde{f}_2}{g_s (1-f_1^2)^{3/2} f_2 h^2} \left(h \tilde{f}_2 f_1^3 g_s' + \tilde{f}_2 f_1^3 g_s h' - h \tilde{f}_2 f_1 g_s' - \tilde{f}_2 f_1 g_s h' - f_1^3 g_s h \tilde{f}_2' \right. \\
& \left. + \tilde{f}_2 f_1^2 g_s h f_1' + f_1 g_s h \tilde{f}_2' + \tilde{f}_2 g_s h f_1' \right) \mathcal{K}_1' - \frac{18K' \tilde{f}_2^2 f_1^2}{f_2 (1-f_1^2)^{1/2} h f_3} F' \\
& + \frac{2f_1^2}{h^2 f_2 f_3^2 (1-f_1^2)^{1/2}} \left(9\tilde{f}_2^2 f_3' h K' f_3 - 2g_s K \right) F + \frac{(9f_3^2 h \mathcal{K}_1 - 2g_s \mathcal{K}_2 K) f_1^2}{h^2 f_2 (1-f_1^2)^{1/2} f_3^2}, \tag{3.5}
\end{aligned}$$

$$\begin{aligned}
0 = & -\omega^2 \mathcal{K}_2 + f_1^2 k^2 \mathcal{K}_2 - \frac{9\tilde{f}_2^2 f_1^2}{h(1-f_1^2)^{1/2} f_2} \mathcal{K}_2'' \\
& + \frac{9\tilde{f}_2 f_1}{g_s (1-f_1^2)^{3/2} f_2 h^2} \left(h \tilde{f}_2 f_1^3 g_s' - \tilde{f}_2 f_1^3 g_s h' + f_1^3 g_s h \tilde{f}_2' - \tilde{f}_2 f_1^2 g_s h f_1' - f_1 g_s h \tilde{f}_2' \right. \\
& \left. - h \tilde{f}_2 f_1 g_s' + \tilde{f}_2 f_1 g_s h' - \tilde{f}_2 g_s h f_1' \right) \mathcal{K}_2' + \frac{9(2f_3^2 g_s h \mathcal{K}_2 - K \mathcal{K}_1 + 4f_3 g_s h F) f_1^2}{2h^2 f_2 (1-f_1^2)^{1/2} g_s f_3^2}. \tag{3.6}
\end{aligned}$$

To make use of the results in [14, 18] we use a radial coordinate x as in (2.36). The physical fluctuations described by (3.4)-(3.6) must satisfy an incoming wave boundary condition at the horizon of the KT BH, and be normalizable at the asymptotic $x \rightarrow 0_+$ boundary. Introducing

$$\mathfrak{w} = \frac{\omega}{2\pi T}, \quad \mathfrak{q} = \frac{k}{2\pi T}. \tag{3.7}$$

The former condition implies

$$F = (1-x)^{-i\mathfrak{w}} \hat{F}, \quad \mathcal{K}_1 = (1-x)^{-i\mathfrak{w}} \hat{\mathcal{K}}_1, \quad \mathcal{K}_2 = (1-x)^{-i\mathfrak{w}} \hat{\mathcal{K}}_2, \tag{3.8}$$

with $\{\hat{F}, \hat{\mathcal{K}}_1, \hat{\mathcal{K}}_2\}$ being regular at the horizon, *i.e.*, as $x \rightarrow 1_-$. Given (3.8), the equations of motion for $\{\hat{F}, \hat{\mathcal{K}}_1, \hat{\mathcal{K}}_2\}$ are complex — they become real once we introduce

$$\mathfrak{w} = -i \Omega, \quad \text{Im}(\Omega) = 0. \quad (3.9)$$

Using the asymptotic expansion for the KT BH developed in [18]⁶, the normalizability condition for $\{\hat{F}, \hat{\mathcal{K}}_1, \hat{\mathcal{K}}_2\}$ at the $x \rightarrow 0_+$ boundary translates into the following asymptotic solution⁷

$$\begin{aligned} \hat{F} = & x^{3/4} F_{3,0} + \left(\frac{\sqrt{2}}{32} \left((2\pi T \Omega)^2 + (2\pi T \mathfrak{q})^2 \right) (F_{3,0} k_s + 5F_{3,0} - \mathcal{K}_{1,3,0}) \right. \\ & - \frac{\sqrt{2}}{32} \left((2\pi T \Omega)^2 + (2\pi T \mathfrak{q})^2 \right) F_{3,0} \ln x \Big) x^{5/4} + \left(F_{7,0} + \left(\frac{6}{7} F_{3,0} a_{2,0} \right. \right. \\ & + (2\pi T \Omega)^4 \left(-\frac{89}{18432} F_{3,0} - \frac{23}{18432} F_{3,0} k_s + \frac{13}{18432} \mathcal{K}_{1,3,0} \right) \\ & - \left(\frac{23}{9216} F_{3,0} k_s - \frac{13}{9216} \mathcal{K}_{1,3,0} + \frac{89}{9216} F_{3,0} \right) (2\pi T \Omega)^2 (2\pi T \mathfrak{q})^2 \\ & + \left(-\frac{23}{18432} F_{3,0} k_s + \frac{13}{18432} \mathcal{K}_{1,3,0} - \frac{89}{18432} F_{3,0} \right) (2\pi T \mathfrak{q})^4 \Big) \ln x \\ & + \left(\frac{1}{1024} (2\pi T \Omega)^2 (2\pi T \mathfrak{q})^2 F_{3,0} + \frac{1}{2048} (2\pi T \Omega)^4 F_{3,0} + \frac{1}{2048} (2\pi T \mathfrak{q})^4 F_{3,0} \right) \ln^2 x \Big) x^{7/4} \\ & + \mathcal{O}(x^{9/4} \ln^3 x), \end{aligned} \quad (3.10)$$

$$\hat{\mathcal{K}}_1 = x^{3/4} \left(\mathcal{K}_{1,3,0} + \frac{1}{2} F_{3,0} \ln x \right) + \mathcal{O}(x^{5/4} \ln^2 x), \quad (3.11)$$

$$\hat{\mathcal{K}}_2 = x^{3/4} \left(\frac{3}{2} \mathcal{K}_{1,3,0} - F_{3,0} + \frac{3}{4} F_{3,0} \ln x \right) + \mathcal{O}(x^{5/4} \ln^2 x), \quad (3.12)$$

where we presented the expansions only to leading order in the normalizable UV coefficients

$$\left\{ F_{3,0}, F_{7,0}, \mathcal{K}_{1,3,0} \right\}. \quad (3.13)$$

The independent UV normalizable coefficients (3.13) imply that the spontaneous χ SB in cascading plasma is associated with the development of the expectation values of the two dimension-3 operators — the gaugino bilinears of the two gauge groups —

$$\mathcal{O}_3^1 \propto F_{3,0}, \quad \mathcal{O}_3^2 \propto \mathcal{K}_{1,3,0}, \quad (3.14)$$

⁶As explained in [18] we can set in numerical analysis $a_0 = 1$.

⁷For numerical analysis we developed all expansions to order $\mathcal{O}(x^{11/4} \ln^5 x)$ inclusive.

and a certain dimension-7 operator⁸

$$\mathcal{O}_7 \propto F_{7,0}. \quad (3.15)$$

Since the equations of motion (3.4)-(3.6) are homogeneous, without the loss of generality we can set $\hat{F}(1) = 1$. The IR, *i.e.*, as $y \equiv (1 - x) \rightarrow 0_+$, asymptotic expansion then takes form⁹

$$\hat{F} = 1 + \mathcal{O}(y^2), \quad \hat{\mathcal{K}}_1 = \mathcal{K}_{1,0}^h + \mathcal{O}(y^2), \quad \hat{\mathcal{K}}_2 = \mathcal{K}_{2,0}^h + \mathcal{O}(y^2), \quad (3.16)$$

where we presented the expansions only to leading order in the normalizable IR coefficients

$$\left\{ \mathcal{K}_{1,0}^h, \mathcal{K}_{2,0}^h \right\}. \quad (3.17)$$

Notice that altogether we have 5 adjustable parameters: (3.13) and (3.17), in order to solve a boundary value problem for a system of 3 second-order differential equations for $\{\hat{F}, \hat{\mathcal{K}}_1, \hat{\mathcal{K}}_2\}$. As a result, a solution produces a dispersion relation for the χ SB quasinormal modes:

$$\Omega = \Omega(\mathbf{q}^2). \quad (3.18)$$

The quasinormal modes signal an instability in plasma provided

$$\text{Im}(\mathfrak{w}) > 0 \Leftrightarrow \Omega < 0, \quad \text{provided} \quad \text{Im}(\mathbf{q}) = 0. \quad (3.19)$$

The results of the analysis of the dispersion relation of χ SB quasinormal modes are presented in Figures 1-2. In principle, we expect discrete branches of the quasinormal modes distinguished by the number of nodes in radial profiles $\{\hat{F}, \hat{\mathcal{K}}_1, \hat{\mathcal{K}}_2\}$. In what follows we consider only the lowest quasinormal mode, which has monotonic radial profiles.

In Figure 1 we study the dispersion relation of the χ SB quasinormal modes at high temperatures. Here, we expect the KT BH to be stable with respect to such fluctuations. Indeed we find that that fluctuations with $(\Omega = 0, \mathbf{q}^2)$ (solid blue line) have $\mathbf{q}^2 < 0$ — as a result, they are massive. The red dashed line

$$\mathbf{q}^2 \Big|_{\text{red,dashed}} = -1.33(7) + 3.62(9) \ln^{-1} \frac{T}{\Lambda} + \mathcal{O} \left(\ln^{-2} \frac{T}{\Lambda} \right), \quad (3.20)$$

⁸It is difficult to identify precisely what is this operator on the gauge theory side of the correspondence. We expect that this operator is not chiral, see section 4.2.2 for more details.

⁹For numerical analysis we developed all expansions to order $\mathcal{O}(y^6)$ inclusive.

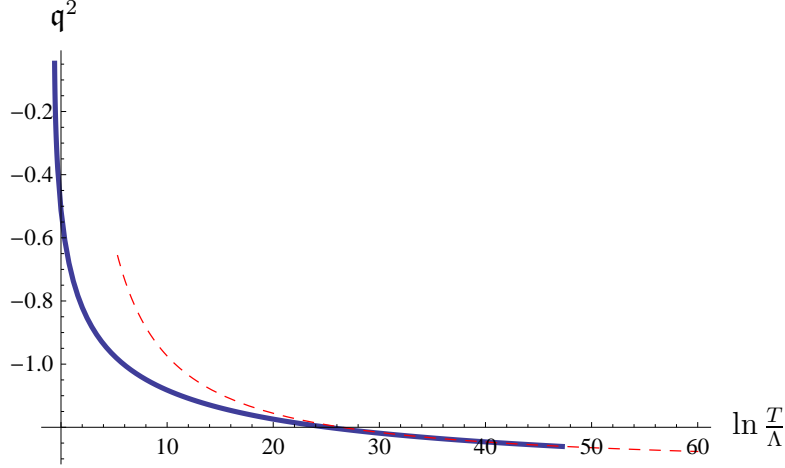


Figure 1: (Colour online) Dispersion relation of the χ SB quasinormal modes of the Klebanov-Tseytlin black hole as a function of $\ln \frac{T}{\Lambda}$ at high temperature. The solid blue line represent the dispersion relation of the χ SB fluctuations with $(i\omega = 0, \mathbf{q}^2)$. The red dashed line is a fit (3.20) to the data.

represents the best fit to (the high-temperature tail of) the data. Notice that in the limit $T \gg \Lambda$ the cascading theory approaches a conformal theory with temperature being the only relevant scale, thus, in agreement with (3.20), \mathbf{q}^2 must approach a constant in this limit.

Figure 2 presents results for the χ SB modes dispersion relation at low temperatures. The solid blue line on the left plot represents the dispersion relation of the χ SB fluctuations at the threshold of instability, *i.e.*, we have $\Omega = 0$ with $\mathbf{q}^2 \neq 0$. Notice that as long as $T > T_{\chi\text{SB}}$ (represented by a vertical blue dashed line) $\mathbf{q}^2 < 0$ for these modes, which makes them massive. As a result, translationary invariant KT horizons are stable against chiral symmetry breaking fluctuations all the way down to $T_{\chi\text{SB}}$. $T_{\chi\text{SB}}$ is lower than the temperature T_c of the confinement/deconfinement phase transition in the cascading plasma (represented by a vertical green dashed line), but is above the temperature T_u (represented by a vertical red dashed line) of the hydrodynamic instability in the deconfined cascading plasma. At temperatures $T < T_{\chi\text{SB}}$ the χ SB fluctuations have $\mathbf{q}^2 > 0$ — they are tachyonic. The right plot on Figure 2 presents detailed dispersion relation in the vicinity of χ SB instability. Here, again, the solid blue line represents the dispersion relation at the threshold of instability ($i\omega = 0, \mathbf{q}^2$); the vertical dashed blue line defines the temperature of the χ SB in the deconfined

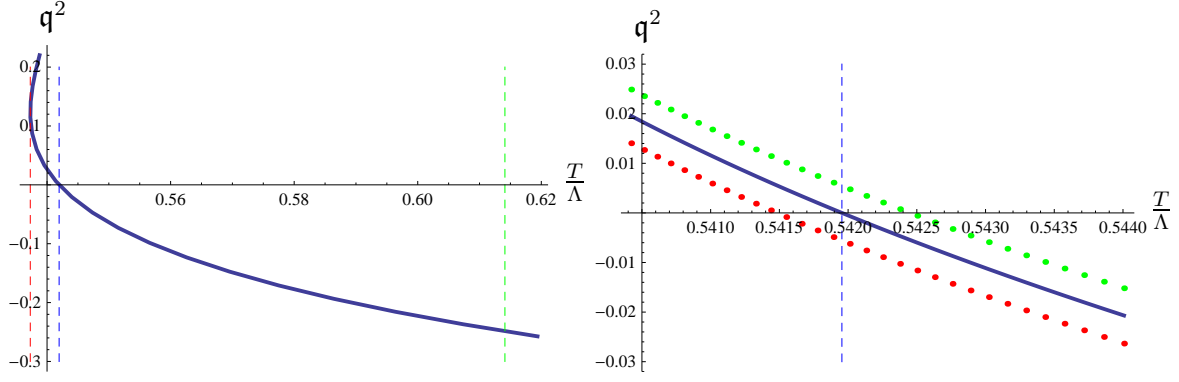


Figure 2: (Colour online) Dispersion relation of the χ SB quasinormal modes of the Klebanov-Tseytlin black hole as a function of $\frac{T}{\Lambda}$. The solid blue lines represent the dispersion relation of the χ SB fluctuations at the threshold of instability: $(\mathfrak{w} = 0, \mathfrak{q}^2)$. The blue dashed vertical lines represent the onset of instability: $T = T_{\chi\text{SB}}$, such that $(i\mathfrak{w} = 0, \mathfrak{q}^2 = 0)$. The vertical dashed green and red lines indicate $T = T_c$ and $T = T_u$ correspondingly. The green dots indicate quasinormal modes with $(i\mathfrak{w} = -0.01, \mathfrak{q}^2)$ as a function of $\frac{T}{\Lambda}$. The red dots indicate quasinormal modes with $(i\mathfrak{w} = 0.01, \mathfrak{q}^2)$ as a function of $\frac{T}{\Lambda}$.

cascading plasma:

$$\left. \frac{T}{\Lambda} \right|_{\chi\text{SB}} = 0.54195(5). \quad (3.21)$$

At a given temperature, quasinormal modes with \mathfrak{q}^2 below the momenta of the modes at the threshold of instability (blue line) are expected to have $i\mathfrak{w} > 0$ (indicating a genuine tachyonic instability), while modes with \mathfrak{q}^2 above the momenta of the modes at the threshold of instability are expected to have $i\mathfrak{w} < 0$ (indicating stable excitations). This is precisely what we find: the red dots on the right plot have $i\mathfrak{w} = 0.01$ and the green dots indicate quasinormal modes with $i\mathfrak{w} = -0.01$.

4 Homogeneous and isotropic end point of chiral tachyon condensation

In section 3 we showed that translationary invariant KT horizons describing the equilibrium states of the deconfined cascading plasma at strong coupling become unstable with respect to the chiral symmetry breaking at $T < T_{\chi\text{SB}}$. In this section we ask

whether the endpoint of the χ SB tachyon condensation in deconfined cascading plasma realizes a homogeneous and isotropic equilibrium state. In other words, we attempt to construct a Klebanov-Strassler black hole (black brane) solution with spontaneously broken chiral symmetry. We present (a technical) evidence that such solution does not exist. A physical argument for the absence of homogeneous and isotropic deconfined equilibrium states in the cascading plasma with spontaneously broken chiral symmetry is presented in section 5¹⁰.

This section is organized as follows. We introduce parametrization of the (generically mass-deformed¹¹) KS BH closely resembling that of KT BH of [18] in section 4.1. The general UV asymptotics obtained from solving equations of motion (2.23)-(2.32) are given in section 4.2. We further identify normalizable and non-normalizable parameters of the UV asymptotics. In particular, we identify two independent mass parameters dual to the gaugino masses in the cascading gauge theory. Thus, a generic solution of (2.23)-(2.32) with a regular Schwarzschild horizon represents a homogeneous and isotropic equilibrium state in *mass-deformed* cascading gauge theory plasma. Because such masses are introduced for the fermions of $\mathcal{N} = 1$ vector multiplet of cascading gauge theory, they explicitly break both the supersymmetry (at zero temperature) and chiral symmetry. The general IR asymptotics of (2.23)-(2.32) guaranteeing a regular Schwarzschild horizon are presented in section 4.3. In section 4.4 we perform the general parameter counting for the numerical analysis. Finally, in section 4.5, insisting on vanishing mass parameters for the gauginos of the cascading gauge theory, we *deform* KT BH solution in the direction of the χ SB tachyons. For zero mass parameters there is always a solution to (2.23)-(2.32): namely, the KT BH with vanishing chiral condensates. One would expect that a sufficiently large deformation in the *tachyonic direction* would lead to a new solution with non-vanishing chiral condensates. As alluded to earlier, we do not find such new solution.

4.1 Parametrization of the KS BH

Let's fix the radial coordinate as in (2.36)

$$x = 1 - f_1(r), \tag{4.1}$$

¹⁰A reader interested in this physical argument would still need the results of sections 4.1-4.4.

¹¹See below.

and introduce new functions $\{K_1, K_2, K_3, f_a, f_b, f_c, h, g\}$ as follows:

$$\begin{aligned}
h_1(x) &= \frac{1}{P} \left(\frac{1}{12} K_1(x) - 36\Omega_0 \right), & h_2(x) &= \frac{P}{18} K_2(x), \\
h_3(x) &= \frac{1}{P} \left(\frac{1}{12} K_3(x) - 36\Omega_0 \right), & g_s(x) &= g(x), \\
\omega_1(x) &= \frac{\sqrt{f_c(x)}}{3(2x-x^2)^{1/4}}, & \omega_2(x) &= \frac{\sqrt{f_a(x)}}{\sqrt{6}(2x-x^2)^{1/4}}, & \omega_3(x) &= \frac{\sqrt{f_b(x)}}{\sqrt{6}(2x-x^2)^{1/4}}, \\
H(x) &= (2x-x^2) h(x).
\end{aligned} \tag{4.2}$$

We can then rewrite equations (2.23)-(2.32) and find that Ω_0 disappears.

4.2 UV asymptotics

The UV asymptotic corresponds to $x \rightarrow 0_+$. We find:

$$K_1 = 4h_0a_0^2 - \frac{1}{2}P^2g_0 - \frac{1}{2}P^2g_0 \ln x + \sum_{n=1}^{\infty} \sum_k k_{1nk} x^{n/4} \ln^k x, \tag{4.3}$$

$$K_2 = 1 + \sum_{n=1}^{\infty} \sum_k k_{2nk} x^{n/4} \ln^k x, \tag{4.4}$$

$$K_3 = 4h_0a_0^2 - \frac{1}{2}P^2g_0 - \frac{1}{2}P^2g_0 \ln x + \sum_{n=1}^{\infty} \sum_k k_{3nk} x^{n/4} \ln^k x, \tag{4.5}$$

$$f_a = a_0 \left(1 + \sum_{n=1}^{\infty} \sum_k f_{ank} x^{n/4} \ln^k x \right), \tag{4.6}$$

$$f_b = a_0 \left(1 + \sum_{n=1}^{\infty} \sum_k f_{bnk} x^{n/4} \ln^k x \right), \tag{4.7}$$

$$f_c = a_0 \left(1 + \sum_{n=2}^{\infty} \sum_k f_{cnk} x^{n/4} \ln^k x \right), \tag{4.8}$$

$$h = h_0 - \frac{P^2g_0}{8a_0^2} \ln x + \sum_{n=2}^{\infty} \sum_k h_{nk} x^{n/4} \ln^k x, \tag{4.9}$$

$$g = g_0 \left[1 + \sum_{n=2}^{\infty} \sum_k g_{nk} x^{n/2} \ln^k x \right]. \tag{4.10}$$

We developed UV expansion to order $n = 8$ inclusive. The expansion depends on 5 microscopic parameters

$$\{g_0, a_0, h_0, k_{110}, f_{a10}\}, \quad (4.11)$$

where g_0 is related to the dimensionless parameter of the cascading gauge theory, and as we explain below, the four independent combinations of the other parameters are related to the temperature, the dynamical scale of the cascading gauge theory, and the two mass parameters (the couplings of the two dimension-3 operators that explicitly break the chiral symmetry of the cascading theory). Besides (4.11), the expansions (4.3)-(4.10) are characterized by 7 vev's:

- those of dimension-3 operators:

$$\{f_{a30}, k_{230}\}, \quad (4.12)$$

- those of dimension-4 operators:

$$\{f_{a40}, g_{40}\}, \quad (4.13)$$

- that of a dimension-6 operator:

$$\{f_{a60}\}, \quad (4.14)$$

- that of a dimension-7 operator:

$$\{f_{a70}\}, \quad (4.15)$$

- and finally, that of a dimension-8 operators:

$$\{f_{a80}\}. \quad (4.16)$$

Note that characterization in (4.12)-(4.16) is suggestive only — typically, a combination of operators is mapped to a given normalizable mode of a dual gravitational field. In what follows we will not need the precise map between the operator vevs and corresponding normalizable coefficients of the dual geometry¹².

¹²The precise map for dimension-4 operators of chirally symmetric states of the cascading gauge theory is discussed in [11].

4.2.1 Mass-deformed KS solution at $T = 0$

In order to understand the physical meaning of the microscopic parameters (4.11) we develop the asymptotic solution of (2.23)-(2.32) in the limit $T = 0$, *i.e.*, for

$$f_1 \equiv 1. \quad (4.17)$$

Here, the radial coordinate (4.1) is undefined, so instead we fix the gauge as

$$\tilde{f}_2 = \frac{r}{3}. \quad (4.18)$$

Similarly to (4.2), we introduce

$$\begin{aligned} h_1(r) &= \frac{1}{P} \left(\frac{1}{12} \tilde{K}_1(r) - 36\Omega_0 \right), & h_2(r) &= \frac{P}{18} \tilde{K}_2(r), \\ h_3(r) &= \frac{1}{P} \left(\frac{1}{12} \tilde{K}_3(r) - 36\Omega_0 \right), & g_s(r) &= \tilde{g}(r), \\ \omega_1(r) &= \frac{\sqrt{\tilde{f}_c(r)}}{3r}, & \omega_2(r) &= \frac{\sqrt{\tilde{f}_a(x)}}{\sqrt{6}r}, & \omega_3(r) &= \frac{\sqrt{\tilde{f}_b(r)}}{\sqrt{6}r}, \\ H(x) &= r^4 \tilde{h}(r), \end{aligned} \quad (4.19)$$

and find that Ω_0 disappears from (2.23)-(2.32).

The UV asymptotic corresponds to $r \rightarrow 0_+$. We find:

$$\tilde{K}_1 = 4\tilde{h}_0 - \frac{1}{2}P^2\tilde{g}_0 - 2P^2\tilde{g}_0 \ln r + \sum_{n=1}^{\infty} \sum_k \tilde{k}_{1nk} r^n \ln^k r, \quad (4.20)$$

$$\tilde{K}_2 = 1 + \sum_{n=1}^{\infty} \sum_k \tilde{k}_{2nk} r^n \ln^k r, \quad (4.21)$$

$$\tilde{K}_3 = 4\tilde{h}_0 - \frac{1}{2}P^2\tilde{g}_0 - 2P^2\tilde{g}_0 \ln r + \sum_{n=1}^{\infty} \sum_k \tilde{k}_{3nk} r^n \ln^k r, \quad (4.22)$$

$$\tilde{f}_a = 1 + \sum_{n=1}^{\infty} \sum_k \tilde{f}_{ank} r^n \ln^k r, \quad (4.23)$$

$$\tilde{f}_b = 1 + \sum_{n=1}^{\infty} \sum_k \tilde{f}_{bnk} r^n \ln^k r, \quad (4.24)$$

$$\tilde{f}_c = 1 + \sum_{n=2}^{\infty} \sum_k \tilde{f}_{cnk} r^n \ln^k r, \quad (4.25)$$

$$\tilde{h} = \tilde{h}_0 - \frac{1}{2}P^2\tilde{g}_0 \ln r + \sum_{n=2}^{\infty} \sum_k \tilde{h}_{nk} r^n \ln^k r, \quad (4.26)$$

$$\tilde{g} = \tilde{g}_0 \left[1 + \sum_{n=2}^{\infty} \sum_k \tilde{g}_{nk} r^n \ln^k r \right]. \quad (4.27)$$

Here, the expansion depends on 4 microscopic parameters:

- the asymptotic string coupling \tilde{g}_0
- the two mass deformation parameters $\tilde{k}_{110}, \tilde{f}_{a10}$
- and the parameter \tilde{h}_0 , related to the strong coupling scale of the cascading gauge theory, see [11]. Given a set $\{\tilde{g}_0, \tilde{k}_{110}, \tilde{f}_{a10}, \tilde{h}_0\}$ the UV solution is uniquely determined by the condensates of various relevant and irrelevant operators, in analogy with (4.12)-(4.16):

$$\{\tilde{f}_{a30}, \tilde{k}_{230}, \tilde{f}_{a40}, \tilde{g}_{40}, \tilde{f}_{a60}, \tilde{f}_{a70}, \tilde{f}_{a80}\}. \quad (4.28)$$

We emphasize that while turning on the finite temperature the four microscopic parameters of the theory must be kept fixed; on the other hand, the condensates (4.28) will develop a nontrivial temperature dependence.

We would like to match (4.20)-(4.27) with the asymptotic finite-temperature solution (4.3)-(4.10). We require that as $r \rightarrow 0$ (and correspondingly $x \rightarrow 0$) all the corresponding warp factors in the metric should agree to leading order, *i.e.*,

$$\begin{aligned} \lim_{\{r,x\} \rightarrow 0} \frac{r^4 \tilde{h}(r)}{(2x - x^2)h(x)} &= 1, & \lim_{\{r,x\} \rightarrow 0} \frac{\tilde{h}(r)^{1/2} \tilde{f}_{a,b,c}(r)}{h(x)^{1/2} f_{a,b,c}(x)} &= 1, \\ \lim_{\{r,x\} \rightarrow 0} \frac{\tilde{K}_{1,2,3}(r)}{K_{1,2,3}(x)} &= 1, & \lim_{\{r,x\} \rightarrow 0} \frac{\tilde{g}(r)}{g(x)} &= 1. \end{aligned} \quad (4.29)$$

This matching uniquely identifies:

$$\begin{aligned} x &= \frac{1}{2}a_0^2 r^4 + \text{higher order}, & g_0 &= \tilde{g}_0, & h_0 a_0^2 &= \tilde{h}_0 + \frac{1}{8}P^2 \tilde{g}_0 \ln\left(\frac{a_0^2}{2}\right), \\ k_{110} &= \frac{2^{1/4}}{a_0^{1/2}} \tilde{k}_{110}, & f_{a10} &= \frac{2^{1/4}}{a_0^{1/2}} \tilde{f}_{a10} + \frac{2^{1/4}}{a_0^{1/2}} \frac{2\tilde{k}_{110} + P^2 \tilde{g}_0 \tilde{f}_{a10}}{3P^2 \tilde{g}_0 + 8\tilde{h}_0} \ln\left(\frac{a_0^2}{2}\right). \end{aligned} \quad (4.30)$$

From (4.30) we see that keeping the microscopic parameters $\{\tilde{g}_0, \tilde{h}_0, \tilde{k}_{110}, \tilde{f}_{a10}\}$ of the mass-deformed cascading gauge theory fixed requires that at finite temperature the following four (corresponding) combinations of (4.11) must be kept fixed

$$\left\{ g_0, \left[h_0 a_0^2 - \frac{1}{8}P^2 g_0 \ln\left(\frac{a_0^2}{2}\right) \right], \left[k_{110} a_0^{1/2} \right], \left[f_{a10} - \frac{2k_{110} + P^2 g_0 f_{a10}}{3P^2 g_0 + 8h_0 a_0^2} \ln\left(\frac{a_0^2}{2}\right) \right] a_0^{1/2} \right\}. \quad (4.31)$$

4.2.2 Comparison with the extremal KS solution

To compare the asymptotic expansion (4.20)-(4.27) with the extremal KS solution (2.34) we need to turn off both the mass deformation parameters

$$\tilde{k}_{110} = \tilde{f}_{a10} = 0. \quad (4.32)$$

We further denote the radial coordinate of the extremal KS solution (2.34), (2.35) as r_{KS} . Then with

$$e^{-r_{KS}} \equiv \frac{3}{8} \sqrt{6} \epsilon^2 r^3 (1 + \mathcal{O}(r^{12} \ln r)) , \quad (4.33)$$

we can identify asymptotic expansions (4.20)-(4.27) with (2.34), (2.35) to order $\mathcal{O}(r^9)$ provided:

$$\begin{aligned} \tilde{h}_0 &= \frac{1}{24} P^2 g_0 (10 \ln(2) - 6 \ln(3) - 4 \ln(\epsilon^2) - 1) , & \tilde{g}_0 &= g_0 , \\ \tilde{f}_{a40} &= \tilde{g}_{40} = \tilde{f}_{a70} = \tilde{f}_{a80} = 0 , \\ \tilde{f}_{a30} &= \frac{3}{4} \sqrt{6} \epsilon^2 , & \tilde{k}_{230} &= -\frac{3}{8} \sqrt{6} \epsilon^2 (5 \ln(2) - 3 \ln(3) - 2 \ln(\epsilon^2)) , \\ \tilde{f}_{a60} &= -\frac{9}{16} \epsilon^4 (-3 + 5 \ln(2) - 3 \ln(3) - 2 \ln(\epsilon^2)) . \end{aligned} \quad (4.34)$$

Notice that in a supersymmetric ground state of the cascading gauge theory (a Klebanov-Strassler solution) the expectation value of dimension-7 operator $\mathcal{O}_7 \propto \tilde{f}_{a70}$ vanishes. It is thus suggestive that this operator is not chiral.

A further rescaling of the radial coordinate (4.18)

$$r \rightarrow \hat{r} = r \epsilon^{2/3} , \quad (4.35)$$

would modify the asymptotic UV-parameters

$$\begin{aligned} &\{\tilde{g}_0, \tilde{h}_0, \tilde{k}_{230}, \tilde{f}_{a30}, \tilde{f}_{a40}, \tilde{f}_{a60}, \tilde{f}_{a70}, \tilde{f}_{a80}, \tilde{g}_{40}\} , \\ &\quad \Downarrow \\ &\{\hat{g}_0, \hat{h}_0, \hat{k}_{230}, \hat{f}_{a30}, \hat{f}_{a40}, \hat{f}_{a60}, \hat{f}_{a70}, \hat{f}_{a80}, \hat{g}_{40}\} , \end{aligned} \quad (4.36)$$

in such a way that UV-parameters of the KS solution (4.34) would take a particularly simple form (note that any reference to the KS scale parameter ϵ , as in (2.34),

disappears):

$$\begin{aligned}
e^{-r_{KS}} &= \frac{3}{8} \sqrt{6} \hat{r}^3 (1 + \mathcal{O}(\hat{r}^{12} \ln(\hat{r}))) , \\
\hat{h}_0 &= \frac{1}{24} P^2 g_0 (10 \ln(2) - 6 \ln(3) - 1) , \quad \hat{g}_0 = g_0 , \\
\hat{f}_{a40} &= \hat{g}_{40} = \hat{f}_{a70} = \hat{f}_{a80} = 0 , \\
\hat{f}_{a30} &= \frac{3}{4} \sqrt{6} , \quad \hat{k}_{230} = -\frac{15}{8} \sqrt{6} \ln(2) + \frac{9}{8} \sqrt{6} \ln(3) , \\
\hat{f}_{a60} &= \frac{27}{16} - \frac{45}{16} \ln(2) + \frac{27}{16} \ln(3) .
\end{aligned} \tag{4.37}$$

4.2.3 Comparison with KT BH

It is straightforward to relate the UV parameters of the KS BH (4.11)-(4.16) with that of the KT BH [18]. First, we need to set

$$k_{110} = f_{a10} = f_{a30} = k_{230} = f_{a70} = 0 . \tag{4.38}$$

The radial coordinate (4.1) is identical to the one used in [18]. Thus, relating the asymptotic expansions (4.3)-(4.10) with the corresponding expressions in [18] we find:

$$h_0 = h_{0,0} , \tag{4.39}$$

$$g_{40} = \frac{g_{2,0}}{g_0} , \tag{4.40}$$

$$f_{a40} = -\frac{1}{7} \frac{a_{2,0}}{a_0} , \tag{4.41}$$

$$f_{a60} = -\frac{1}{4} \frac{a_{3,0}}{a_0} , \tag{4.42}$$

$$\begin{aligned}
f_{a80} &= \frac{1}{\delta} \left\{ P^2 g_0 \left(\frac{6366}{245} \left(\frac{a_{2,0}}{a_0} \right)^2 - 6 \left(\frac{g_{2,0}}{g_0} \right)^2 + 6 + \frac{74}{7} \frac{a_{2,0}}{a_0} - 41 \frac{a_{4,0}}{a_0} \right) + h_{0,0} \left(\frac{480}{7} \frac{a_{2,0}}{a_0} \right. \right. \\
&\quad \left. \left. + \frac{126144}{245} \left(\frac{a_{2,0}}{a_0} \right)^2 - 120 \frac{a_{4,0}}{a_0} - \frac{576}{7} \frac{a_{2,0}}{a_0} \frac{g_{2,0}}{g_0} \right) + \frac{13824}{49} \left(\frac{a_{2,0}}{a_0} \right)^2 \frac{h_{0,0}^2}{P^2 g_0} \right\} ,
\end{aligned} \tag{4.43}$$

where

$$\delta = 139 P^2 g_0 - 120 h_{0,0} a_0^2 . \tag{4.44}$$

4.3 IR asymptotics

Introducing $y = 1 - x$, the regular horizon $y \rightarrow 0_+$ asymptotics of

$$\{K_1, K_2, K_3, f_a, f_b, f_c, h, g\},$$

(defined as in (4.2)) take form:

$$\begin{aligned} K_i &= \sum_{n=0}^{\infty} k_{ihn} y^{2n}, \quad i = 1, 2, 3, \\ f_\alpha &= a_0 \sum_{n=0}^{\infty} f_{\alpha hn} y^{2n}, \quad \alpha = a, b, c, \\ h &= \sum_{n=0}^{\infty} h_{hn} y^{2n}, \quad g = g_0 \sum_{n=0}^{\infty} g_{hn} y^{2n}. \end{aligned} \tag{4.45}$$

We developed IR expansion to order $n = 1$ inclusive. Here, the expansion is characterized by 9 parameters:

$$\{k_{1h0}, k_{2h0}, k_{3h0}, f_{ah0}, f_{ah1}, f_{bh0}, f_{ch0}, h_{h0}, g_{h0}\}. \tag{4.46}$$

4.3.1 Comparison with KT BH

By matching the near-horizon asymptotic expansions (4.45) with the corresponding ones in [18], we can relate (4.46) to those of the KT BH solution. We find:

$$\begin{aligned} h_{h0} &= h_0^h, \quad g_{h0} = \frac{g_0^h}{g_0}, \quad k_{1h0} = k_{3h0} = k_0^h, \quad k_{2h0} = 1, \\ f_{ah0} &= f_{bh0} = \frac{b_0^h}{a_0}, \quad f_{ch0} = \frac{a_0^h}{a_0}, \\ f_{ah1} &= \frac{1}{a_0 \delta_h} \left[\left(2h_0^h a_0^h (3a_0^h + 2a_1^h) - \frac{1}{2} P^2 g_0^h \right) b_0^h + 6h_0^h (a_0^h + 2a_1^h) (b_0^h)^2 \right], \end{aligned} \tag{4.47}$$

where as in [18]

$$\delta_h \equiv 8h_0^h (a_0^h)^2 - P^2 g_0^h. \tag{4.48}$$

4.4 Parameter counting and the numerical procedure

In this section we would like to further understand the physical meaning of the microscopic parameters (4.11). As we mentioned, $g_0 P^2$ is the dimensionless parameter of the cascading theory (which must be large for the gravity approximation to be valid),

while a_0, h_0, k_{110} and f_{a10} are related to the strong coupling scale of cascading theory, the two mass parameters, and to the temperature.

Much like in [18], it can be shown that

$$a_0^2 = 4\pi G_5 sT, \quad (4.49)$$

where T is the temperature of the black hole, s is its entropy density, and the effective five-dimensional Newton's constant G_5 is given by (2.19).

Following [18], we introduce a new dimensionless coefficient k_s as

$$P^2 g_0 k_s = 4h_0 a_0^2 - \frac{1}{2} P^2 g_0. \quad (4.50)$$

The second constraint in (4.31) then implies that the combination $\left[k_s - \frac{1}{2} \ln\left(\frac{a_0^2}{2}\right)\right]$ is independent of the temperature. Thus, we can choose it to define the strong coupling scale Λ of the cascading theory:

$$k_s \equiv \frac{1}{2} \ln\left(\frac{a_0^2}{\Lambda^4}\right) = \frac{1}{2} \ln\left(\frac{4\pi G_5 sT}{\Lambda^4}\right). \quad (4.51)$$

Using the expressions for the high temperature entropy density of the theory computed in [18], we see that at high temperatures $k_s \simeq (1/2) \ln(T^4/\Lambda^4)$, with corrections scaling as $\ln(\ln(T/\Lambda))$. We will use k_s instead of the temperature as our basic dimensionless parameter, and use (4.51) to translate between k_s and T/Λ .

Further introducing K_{110} via the relation

$$k_{110} = K_{110} \left(3P^2 g_0 + 8h_0 a_0^2\right) - \frac{1}{2} P^2 g_0 f_{a10}, \quad (4.52)$$

the remaining constraints in (4.31) are solved with

$$f_{a10} = (\mu_1 + 4\mu_2 k_s) e^{-k_s/2}, \quad K_{110} = \mu_2 e^{-k_s/2}, \quad (4.53)$$

where μ_i are the fixed (reduced) mass-deformation parameters of the cascading gauge theory

$$\mu_i \equiv \frac{m_i}{\Lambda}, \quad m_i = \text{constant}. \quad (4.54)$$

From (4.53) we see that at high temperatures, $T \gg \Lambda$,

$$f_{a10} \simeq \frac{1}{T} \left(m_1 + 8 m_2 \ln\left(\frac{T}{\Lambda}\right)\right), \quad K_{110} \simeq \frac{m_2}{T}. \quad (4.55)$$

Note that our metric ansatz (see (2.22), (4.2)) is invariant under a scaling symmetry taking

$$(t, x_1, x_2, x_3) \rightarrow \lambda^{-1/2} (t, x_1, x_2, x_3), \quad h \rightarrow \lambda^{-2} h, \quad f_{a,b,c} \rightarrow \lambda f_{a,b,c}, \quad (4.56)$$

and leaving all other functions in our solution (as well as the coordinate x) invariant. We can now use the scaling symmetry (4.56) to set

$$a_0 = 1. \quad (4.57)$$

Recall also that we are solving the theory in the supergravity approximation, which includes only the leading order terms both in the g_s expansion and in the curvature (α') expansion. When we neglect g_s corrections, the action (and the equations of motion we wrote) does not depend separately on P^2 and g but only on the combination $P^2 g$. We can thus set $g_0 = 1$, and recall that whenever we have a factor of P^2 we really mean $P^2 g_0$. Furthermore, when we neglect α' corrections, the action is multiplied by a constant when we rescale the ten dimensional metric by a constant factor (and rescale the p -forms accordingly), so that the equations of motion are left invariant; this transformation acts on our variables as

$$h \rightarrow \lambda^2 h, \quad f_{a,b,c} \rightarrow f_{a,b,c}, \quad K_{1,3} \rightarrow \lambda^2 K_{1,3}, \quad K_2 \rightarrow K_2, \quad g \rightarrow g, \quad (4.58)$$

and it changes P by $P \rightarrow \lambda P$. We can use this transformation to relate the solutions for different values of P (as long as we are in the supergravity approximation). Thus, we will perform the numerical analysis for $P = 1$, and we can use (4.58) to obtain the solutions for any other value of P .

We are now ready to formulate our numerical procedure, and count the parameters of the solution:

- We integrate the differential equations along x -coordinate

$$0 \leq x \leq 1, \quad (4.59)$$

with $x = 0$ being the boundary and $x = 1$ being the horizon.

- We use various scaling symmetries discussed above to set

$$P = g_0 = a_0 = 1. \quad (4.60)$$

- Altogether we need to integrate 8 functions

$$\{K_1, K_2, K_3, f_a, f_b, f_c, h, g\}, \quad (4.61)$$

for a given set of the remaining microscopic parameters $\{k_s, f_{a10}, K_{110}\}^{13}$.

■ The solution is then determined by 7 UV parameters (4.12)-(4.16), and 9 IR parameters (4.46):

$$\begin{aligned} \text{UV :} & \quad \{f_{a30}, k_{230}, f_{a40}, g_{40}, f_{a60}, f_{a70}, f_{a80}\}, \\ \text{IR :} & \quad \{k_{1h0}, k_{2h0}, f_{3h0}, f_{ah0}, f_{ah1}, f_{bh0}, f_{ch0}, h_{h0}, g_{h0}\}. \end{aligned} \quad (4.62)$$

Overall we have 16 parameters, precisely what is necessary to determine (4.61) from the appropriate second order differential equations.

We follow numerical method introduced in [18]. In a nutshell, for a fixed set of microscopic parameters $\{k_s, f_{a10}, K_{110}\}$, we choose a 'trial' set of parameters (4.62) and integrate (a double set of) the equations of motion for (4.61) from the UV ($x_{\text{initial}} = 0.01$) to $x = 0.5$, and from the IR ($y_{\text{initial}} = 0.01$) to $y = 0.5$. A solution (4.62) of the boundary value problem implies that the mismatch vector

$$\begin{aligned} \vec{v}_{\text{mismatch}} \equiv & \left(K_1^b - K_1^h, (K_1^b + K_1^h)', K_2^b - K_2^h, (K_2^b + K_2^h)', K_3^b - K_3^h, \right. \\ & (K_3^b + K_3^h)', f_a^b - f_a^h, (f_a^b + f_a^h)', f_b^b - f_b^h, (f_b^b + f_b^h)', f_c^b - f_c^h, \\ & \left. (f_c^b + f_c^h)', h^b - h^h, (h^b + h^h)', g^b - g^h, (g^b + g^h)' \right)_{x=y=0.5}, \end{aligned} \quad (4.63)$$

with the superscripts b and h referring to the boundary (UV) and the horizon (IR) integrations, vanishes. At each iteration we adjust the set of parameters (4.62) along the direction of the steepest decent for $\|\vec{v}_{\text{mismatch}}\|$. In practice, for a valid numerical solution we were able to achieve

$$\|\vec{v}_{\text{mismatch}}\| \sim 10^{-13} \dots 10^{-11}. \quad (4.64)$$

4.5 Deformation of KT BH along χ SB tachyonic directions

In section 3 we identified instabilities of the translationary invariant KT BH horizons, provided $T < T_{\chi\text{SB}}$. Earlier in this section we setup a general numerical boundary value problem to determine translationary invariant regular horizon geometries with spontaneously broken chiral $U(1)$ symmetry. Here, we outline our attempts to construct such geometries.

¹³We can always use (4.51), (4.53), and (4.54) to convert these parameters into the physical temperature and the masses.

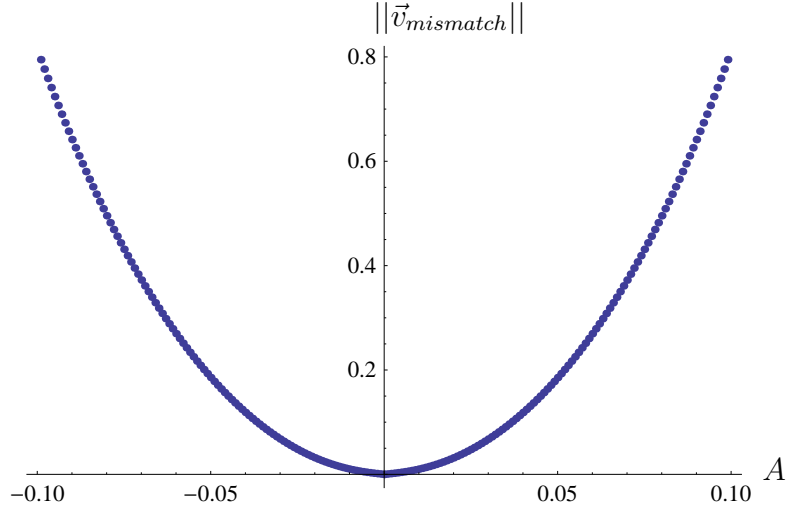


Figure 3: A minimum of the mismatch vector $||\vec{v}_{mismatch}||$ (4.63) as a function of the 'tachyon deformation amplitude' A (4.68) for $A \neq 0$ would identify seed values of parameters (4.62) leading to a homogeneous and isotropic KS BH solution with spontaneously broken chiral symmetry. We use $k_s = -0.8$. Clearly, such minimum is not present.

First of all, since we are interested in *spontaneous* as opposite to *explicit* χ SB we set the mass deformation parameters (see (4.53)) to zero:

$$f_{a10} = K_{110} = 0. \quad (4.65)$$

Second, motivated by the analysis of section 3, we consider values of k_s , such that the temperature of the KT BH is below the temperature of the chiral tachyonic instability $T_{\chi\text{SB}}$, but is still above the temperature of the hydrodynamic instability T_u . This translates into the range

$$k_s^{\text{unstable}} = -1.230(3) < k_s < k_s^{\chi\text{SB}} = 0.77743(2). \quad (4.66)$$

The main difficulty associated with solving the boundary value problem (observed also in the analysis in [14, 18]) is that the basin of attraction of the parameters (4.62) resulting in the convergent iterative process for the steepest decent for the norm of the mismatch vector (4.63) is quite narrow; moreover, it becomes more and more narrow as k_s (or equivalently the temperature) decreases. In other words, to obtain a solution one has to have a pretty good guess for the seed (initial) values of (4.62). Clearly,

having a 16-dimensional parameter space this is a daunting task! Of course, identical problem¹⁴ exists for finding the KT BH solution. It is instructive to recall how this issue was circumvented in [14, 18]:

- from general field theoretic arguments, *i.e.*, high-temperature restoration of the spontaneously broken symmetry, KT BH was supposed to exist at arbitrary high temperatures [5];
- for $T \gg \Lambda$ one can develop an analytic high-temperature solution for the KT BH [17];
- from the analytic high-temperature solution we can extract the values of the parameters for the boundary value problem and use them as 'seeds' [18];
- finally, we can slowly lower the temperature using as 'seeds' parameters obtained from solution of the boundary value problem at previous (slightly higher) temperature.

Rather remarkably, a described procedure, for small enough temperature decrements — typically $\frac{\delta k_s}{k_s} \sim 10^{-2}$ — resulted in convergence of the norm of the mismatch vector from initial values of order 10^{-1} to values (4.64) in 8 or less iterations.

Given that the instability of the KT BH towards generating chiral condensates exists only below certain temperature, there is no high-temperature (analytic) guide for the seed values of (4.62). In the rest of this section we explain one of our unsuccessful attempts to produce KS BH solution. As we emphasized before, in the limit of vanishing masses (4.65), for every values of k_s we should recover the appropriate KT BH solution. Indeed, this is what we found: for instance, for $k_s = -0.8$ we recovered (with precision $\sim 10^{-8}$) KT BH parameters (we need to use (4.39)-(4.44) and (4.47)) with

$$f_{a30} \sim k_{230} \sim f_{a70} \sim 10^{-9}, \quad (4.67)$$

where the exact (expected) values should vanish (4.38)¹⁵. Our idea was to start with a KT BH solution and deform its set of parameters with the parameters of the linearized χ SB tachyon (3.13) and (3.17) at amplitude A (see (2.40)). At the level of functions (4.61),

$$\begin{aligned} K_1 &= K^{KT} + A \delta k_1, & K_2 &= 1 + A \delta k_2, & K_3 &= K^{KT} - A \delta k_1, \\ f_a &= f_3^{KT} + A \delta f, & f_b &= f_3^{KT} - A \delta f, \\ f_c &= f_2^{KT}, & h &= h^{KT}, & g &= g^{KT}, \end{aligned} \quad (4.68)$$

where the fluctuations $\{\delta f, \delta k_1, \delta k_2\}$ (computed at the threshold of instability) are

¹⁴The only difference being that the corresponding parameter space there is 10-dimensional.

¹⁵This provides a highly nontrivial consistency check on our analysis.

substituted with $\mathbf{q} = 0$ ¹⁶. A linearized tachyon deformation (4.68) is off-shell now (since \mathbf{q} should vanish), but it is only slightly off-shell¹⁷ if k_s is close enough to $k_s^{\chi\text{SB}}$. Physically, what we are doing is to allow the KT BH chiral tachyon to roll and build up the χSB condensates of amplitude $\sim A$. The expectation is that as we scan the 'seeds' constructed from (4.68) as a function of A we should reach a new basin of attraction, different from the one of the KT BH solution, in the parameter space (4.62). The iterative procedure in this new basin of attraction (as described at the end of section 4.4) would produce a KS BH solution. A signature of a new basin of attraction would be a minimum of the mismatch vector constructed from the 'seed' parameters from (4.68) as a function of A , for $A \neq 0$. Figure 3 presents typical results of such analysis¹⁸. We used $k_s = -0.8$ for data in Figure 3. The absence of a minimum in $||\vec{v}_{\text{mismatch}}||$ away from $A \neq 0$ is one piece of the evidence that a homogeneous and isotropic KS horizon with spontaneously broken chiral symmetry does not exist. Since χSB fluctuations of the KT horizons are tachyonic for $T < T_{\chi\text{SB}}$, these tachyons must condense with finite momenta, resulting in non-homogeneous and non-isotropic ground state. In section 5 we present independent analysis pointing to the same conclusion.

5 Homogeneous and isotropic states of mass-deformed cascading plasma

We argued in section 3 that chirally symmetric deconfined phase of the cascading plasma becomes unstable with respect to fluctuations spontaneously breaking the chiral symmetry, provided $T < T_{\chi\text{SB}}$. We further presented the evidence in section 4.5 that these tachyons do not condense at zero momentum in a new ground state — in other words, the metastable¹⁹ equilibrium phase of deconfined cascading plasma at $T < T_{\chi\text{SB}}$ breaks the chiral $U(1)$ symmetry spontaneously, but is not homogeneous and isotropic. In this section we present alternative arguments, leading to the same conclusion.

Effective gravitational action (2.20) can describes homogeneous and isotropic ther-

¹⁶We need to substitute $\mathbf{q} = 0$, otherwise the seed functions (4.68) do not describe homogeneous and isotropic horizon.

¹⁷If a homogeneous and isotropic KS BH horizon exists.

¹⁸The norm of mismatch vector very rapidly and monotonically increases to $\sim 10^2$ as $|A|$ increases to 0.3.

¹⁹Recall that $T_{\chi\text{SB}}$ is below the temperature of the first order confinement/deconfinement phase transition in cascading plasma.

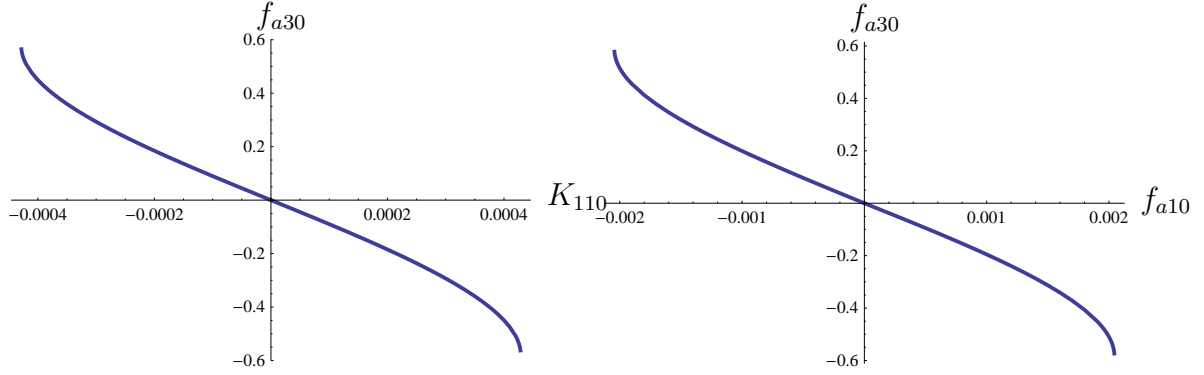


Figure 4: One of the chiral condensates (f_{a30}) in mass-deformed cascading plasma as a function of mass-parameters K_{110} with $f_{a10} = 0$ (left plot) and f_{a10} with $K_{110} = 0$ (right plot) (see (4.53) for the precise relation to gaugino masses) for $k_s = -0.8$. Notice that the condensate vanishing linearly in the chiral limit.

mal states of mass-deformed cascading plasma (see section 4.4). Specifically, we can introduce two independent mass-parameters $\mu_i = \frac{m_i}{\Lambda}$, $i = 1, 2$ related to the non-normalizable coefficients $\{f_{a10}, K_{110}\}$ (see (4.53)) of the general asymptotic UV expansion (4.3)-(4.10) of the holographically dual gravitational background. These are the mass terms for the gauginos of the cascading gauge theory $\mathcal{N} = 1$ vector multiplets. Gaugino mass terms explicitly break chiral symmetry, and thus the thermal state of the mass-deformed cascading gauge theory should exist at arbitrary high temperatures. In particular, as recalled in section 4.5, we can now follow the strategy of constructing mass-deformed KS BH solution by developing first a mass-deformed high-temperature expansion, and then using obtained values of normalizable coefficients as 'seeds' for (4.62). Slowly varying k_s we can reach low temperatures. Finally, we can numerically consider the limit of vanishing masses and study whether or not the condensates $\{f_{a30}, k_{230}, f_{a70}\}$ survive the chiral limit.

We present only the final results²⁰. We consider $k_s = -0.8$, which corresponds to temperatures below the condensation of the χ SB fluctuations, see (4.66). Figure 4 presents results for the chiral condensate f_{a30} (the remaining chiral condensates

²⁰Much like in case of KT BH solution [18], consistency of the 'full solution' and its high-temperature limit for $T \gg \Lambda$ is a highly nontrivial check.

$\{k_{230}, f_{a70}\}$ have identical qualitative behavior) in two special²¹ cases:

$$\begin{aligned} \text{left plot :} & \quad (K_{110} \neq 0, f_{a10} = 0), \\ \text{right plot :} & \quad (K_{110} = 0, f_{a10} \neq 0). \end{aligned} \tag{5.1}$$

The map between the gravitational parameters $\{f_{a10}, K_{110}\}$ and the mass-parameters of the deformed cascading plasma is given by (4.53). Notice that in both cases, in the chiral limit, the condensates vanish linearly with the mass parameter:

$$f_{a30} \propto K_{110} \rightarrow 0, \quad \text{or} \quad f_{a30} \propto f_{a10} \rightarrow 0. \tag{5.2}$$

Thus, we conclude that homogeneous and isotropic states of the deconfined cascading plasma do not break chiral symmetry spontaneously.

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²¹We tried other mass-deformations, and the results are qualitatively identical.

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